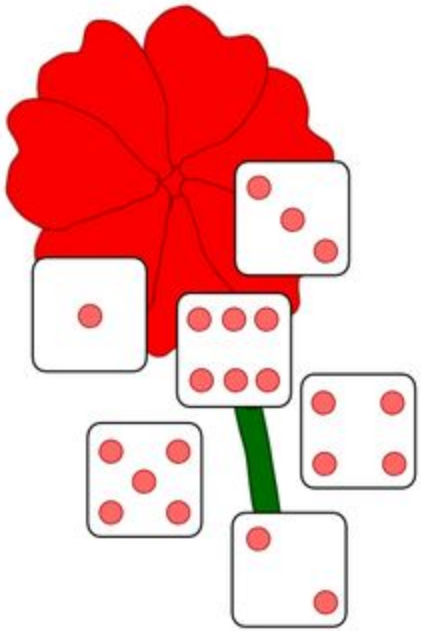
Puzzles and Problem Solving

Duke TIP Academic Adventures

Petals Around the Rose

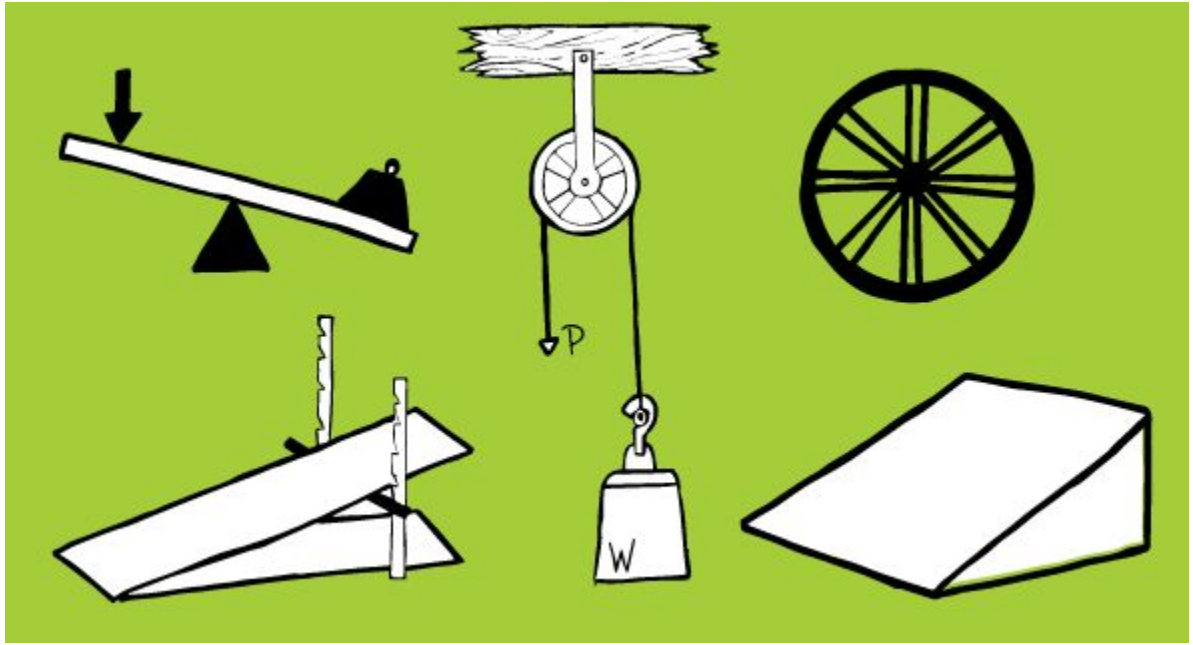


Third Person Introductions

Introduce the person sitting next you by saying the following.

- Name
- Hometown
- Favorite Game or Puzzle

Human Machine



Rules and Expectations

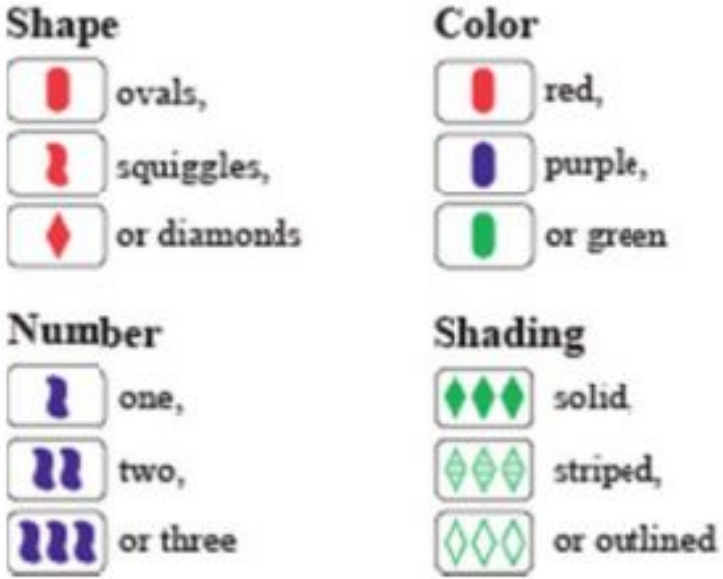
- Be respectful.
- Participate.
- When someone else is talking, you are not.
- Do not shout out answers.
- Have fun!

Finishing Early

Everyone works at a different pace. If you you happen to finish something early, please raise your hand and let me know. You may get to be my assistant!

Warm Up

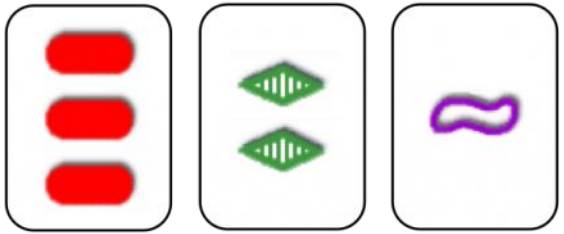
Game of SET



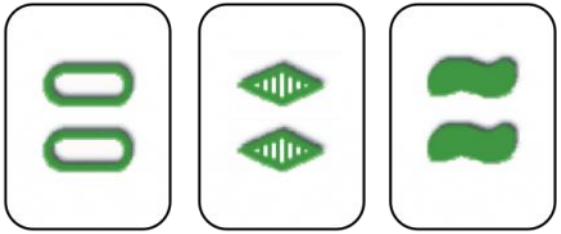
A set consists of 3 cards in which each of the cards features, looked at one-by-one, are the same on each card, or different on each card.

Game of SET

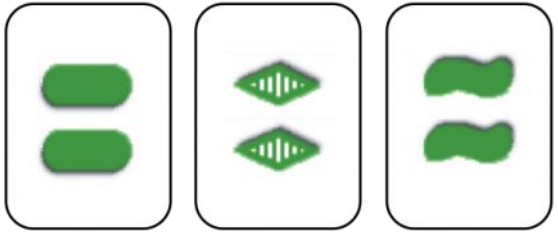
YES



YES

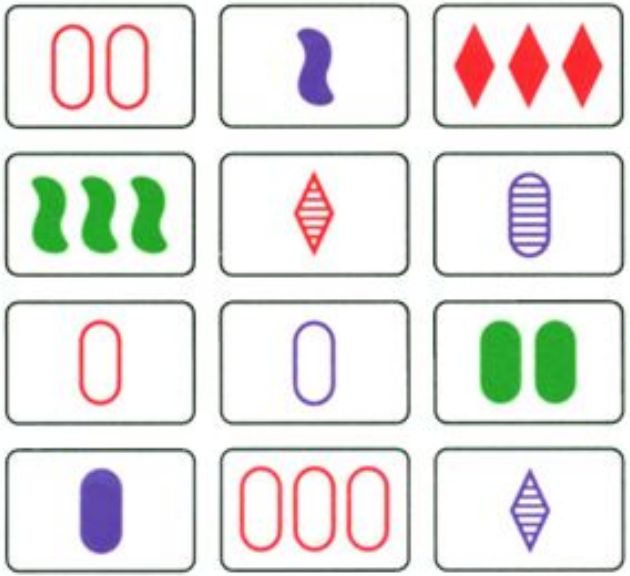


NO



Game of SET

Let’s practice!



Game of SET

- At your table deal 12 cards face up.
- Players do not take turns, but pick up SETs as they see them. If everyone agrees there are no SETs, deal 3 more cards on the table.
- Replace the cards from the top of the deck when a SET is removed.
- The person with the most SETs wins!

Game of SET

1. If there are no duplicates, how many cards are in a deck of SET?

2. What is the probability of drawing a green card?

3. What is the probability of drawing a purple diamond card?

4. What is the probability of drawing a card that is red or has an oval on it?

Game of SET

1. There are

$$3 \times 3 \times 3 \times 3 = 3^4 = 81$$

cards are in a deck of SET.

Game of SET

2. The probability of drawing a green card is

$$27/81 = 1/3$$

Game of SET

3. The probability of drawing a purple diamond card is

$$9/81 = 1/9$$

Game of SET

4. The probability of drawing a card that is red or has an oval on it is

$$(27 + 27 - 9)/81 = 45/81 = 5/9$$

Brain Teaser

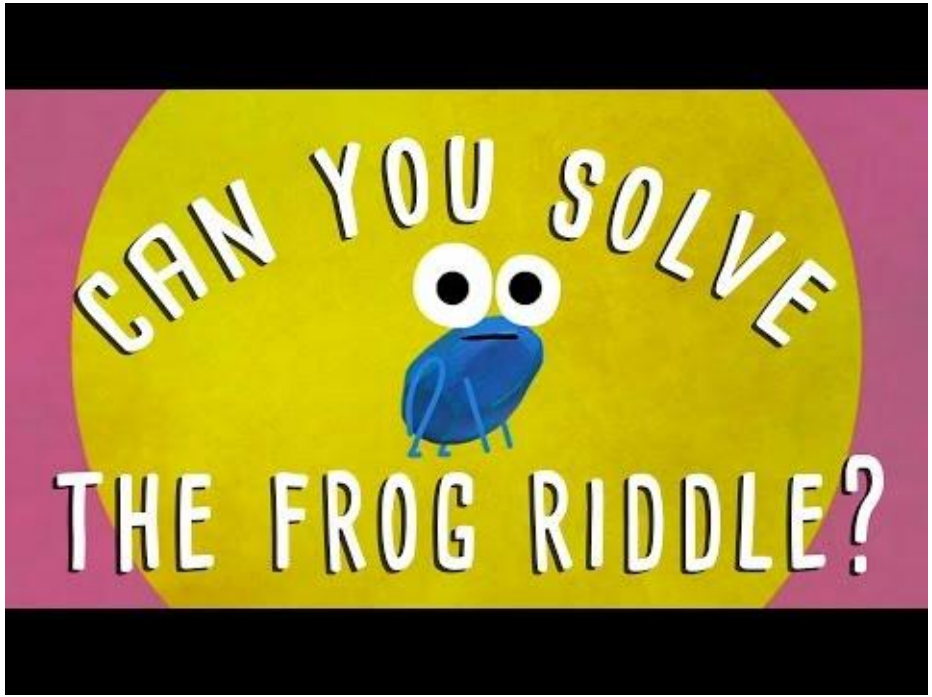
There is a number that is 5 times the sum of its digits. What is this number?

Brain Teaser

The number would be 45 since

$$5(4 + 5) = 5(9) = 45.$$

Brain Teaser



Nims

Rules: Start with a pile or piles of objects. Players take turns removing 1, 2, or 3 objects from 1 pile. The player that removes the last piece loses.

Nims

- Play 1 pile Nims with a partner. Take turns going first.
- Try to come up with a strategy to win every time.
- Once you figure out a winning strategy for 1 pile Nims, start playing 2 pile Nims and do the same.

Nims

- What were some winning strategies for 1 pile Nims?

- How about 2 pile Nims?

Nims

Mr. Patterson and Ms. Ritchey play a game of Nims with 1 pile of 17. How many moves does the shortest game take? How many moves does the longest game take?

Nims

The shortest game takes 6 moves and the longest game takes 17 moves.

Nims

Mr. Patterson and Ms. Ritchey play a game of Nims with 2 piles, with 11 in one pile and 13 in the other.
How many moves does the shortest game take?
How many moves does the longest game take?

Nims

The shortest game takes 9 moves and the longest game takes 24 moves.

Brain Teaser

Mr. Patterson’s sock drawer is a mess! He has 12 black socks and 10 white socks in his drawer, but none of them are matched up. He blindly picks up some socks from his drawer. What is the minimum number of socks Mr. Patterson will have to pick to be certain to find a pair of socks of the same color?

Brain Teaser

3 socks. After picking 1 sock, there is no way Mr. Patterson can have a match. If he picks 2 socks, he may have a match, but could also have 1 black and 1 white sock. After Mr. Patterson picks 3 socks, he is guaranteed a match!

Brain Teaser

What is the minimum number of socks Mr. Patterson will have to pick to be certain to find a pair of black socks? Pair of white socks?

Brain Teaser

12 socks. Mr. Patterson could pick 10 white socks in a row. However, after two more picks, he would be guaranteed to have a pair of black socks.

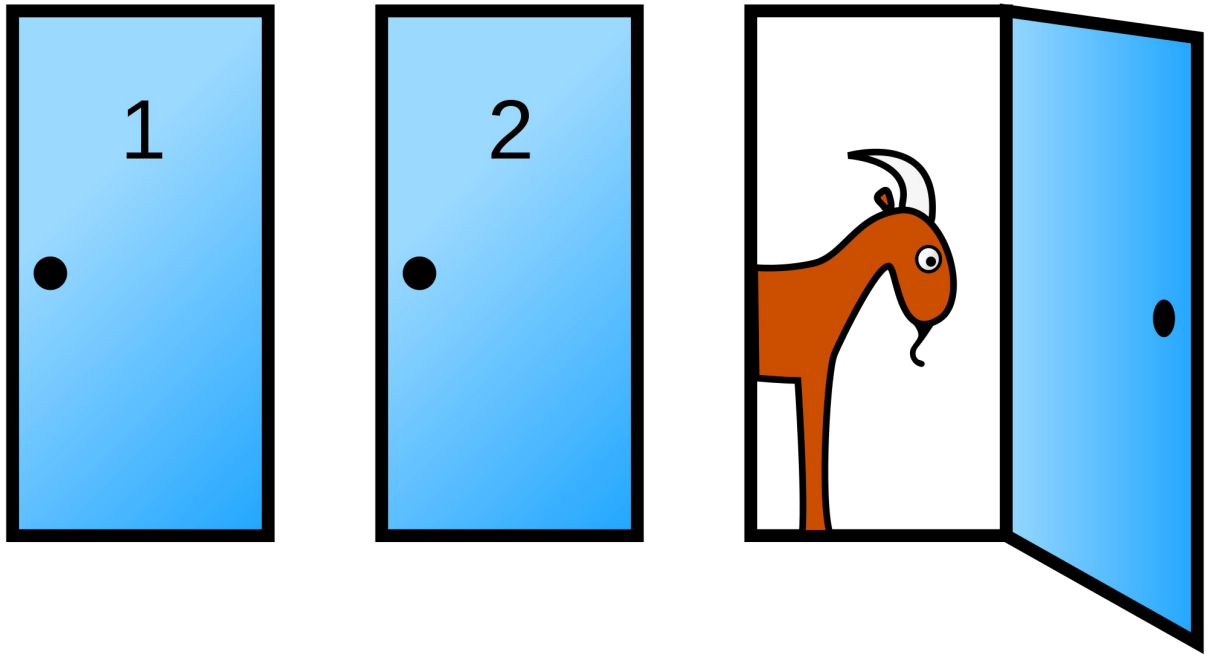
14 socks. Mr. Patterson could pick 12 black socks in a row. However, after two more picks, he would be guaranteed to have a pair of white socks.

Lunch

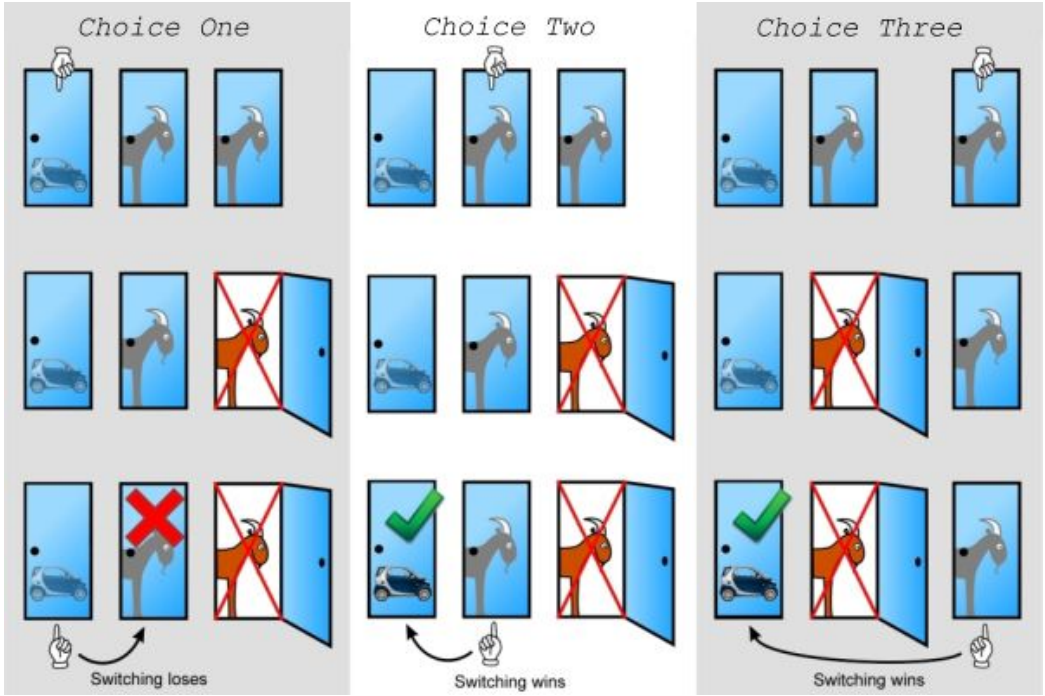
Warm Up

Suppose you are on a game show and you can choose between three doors. Behind one door is a brand new car and behind the other two are old goats. After choosing one of the doors, the host shows you a goat behind one of the doors you did not choose. Should you stay with the original door you choose or switch? Does it make a difference?

Warm Up



Warm Up


Choice One
Choice Two
Choice Three
Switching loses
Switching wins
Switching wins

Magic Squares

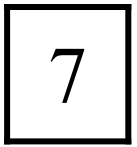
4	9	2
3	5	7
8	1	6

Magic Squares

- Entries must all be different.
- Entries must all be positive whole numbers.
- The sum of all rows, columns, and diagonals must be equal.

Magic Squares

We can create a 1 by 1 magic square



but that’s boring!

Magic Squares

Can we create a 2 by 2 magic square? Why or why not?

Magic Squares

If our magic square is

A	B
C	D

then A + B = A + C, which means B = C. Since all numbers must be different, we cannot create a 2 by 2 magic square.

Magic Squares

- Create your own 3 by 3 magic square. Be sure to check that it works!

- If you can do this, try creating a 4 by 4!

Magic Squares

What strategies did you use to create your magic square?

Magic Squares

Below is a way to create a magic square that will always work.

C - B	C + A + B	C - A
C - A + B	C	C + A - B
C + A	C - A - B	C + B

Magic Squares

7 - 2	7 + 3 + 2	7 - 3
7 - 3 + 2	7	7 + 3 - 2
7 + 3	7 - 3 - 2	7 + 2

5	12	4
6	7	8
10	2	9

Magic Squares

23	6	19	2	15
4	12	26	8	16
10	18	1	14	22
11	24	7	20	3
17	5	13	21	9

Magic Squares

23	6	19	2	15
4	12	26	8	16
10	18	1	14	22
11	24	7	20	3
17	5	13	21	9

Magic Squares

23	6	19	2	15
4	12	25	8	16
10	18	1	14	22
11	24	7	20	3
17	5	13	21	9

Magic Squares

Benjamin Franklin's Magic Square

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

Magic Squares

The magic number is 260!

Brain Teaser

In the park I saw a total of 12 bicycles and tricycles. If the total number of wheels was 28, how many bicycles and tricycles were there?

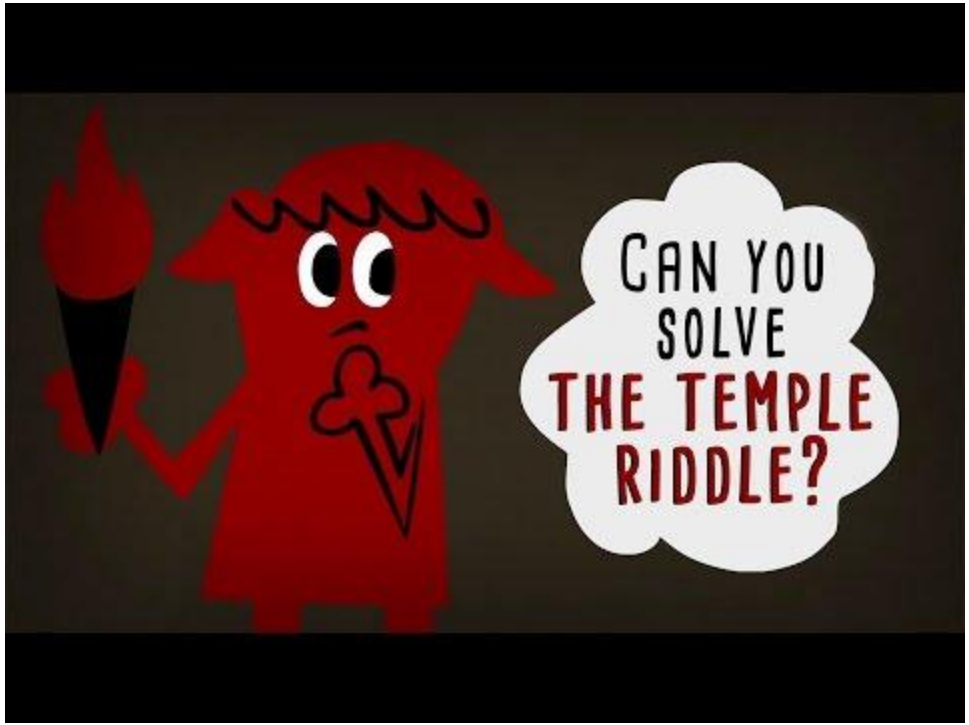
Brain Teaser

There were 8 bicycles and 4 tricycles.

Notice $8 + 4 = 12$, which is the total number and

$2(8) + 3(4) = 16 + 12 = 28$, which is the total number of wheels.

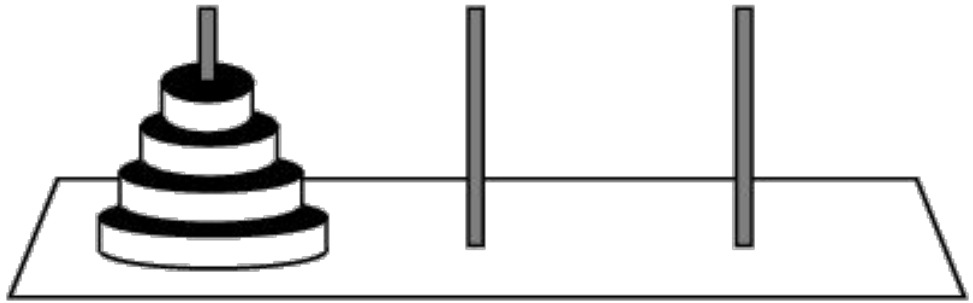
Brain Teaser



Tower of Hanoi

Move entire stack from the left rod to the right rod using the following rules:

1. Only one disk can be moved at a time.
2. Only the top disk can be moved.
3. No disk may be placed on a smaller disk.



Tower of Hanoi

- It takes 1 move to solve the puzzle with 1 disk.

- Try to solve the puzzle starting with 2 disks. How many moves did it take? Record it in a table.

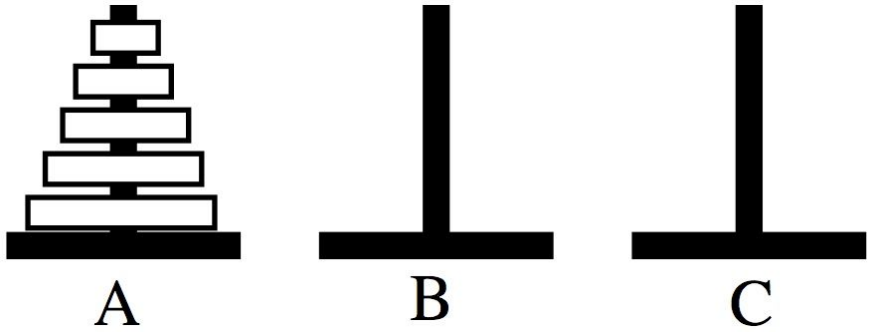
Tower of Hanoi

Try the same with 3, 4, 5, and 6 disks. Can you come up with a strategy to solve the puzzle that always works?

Tower of Hanoi

If the number of pegs is even:

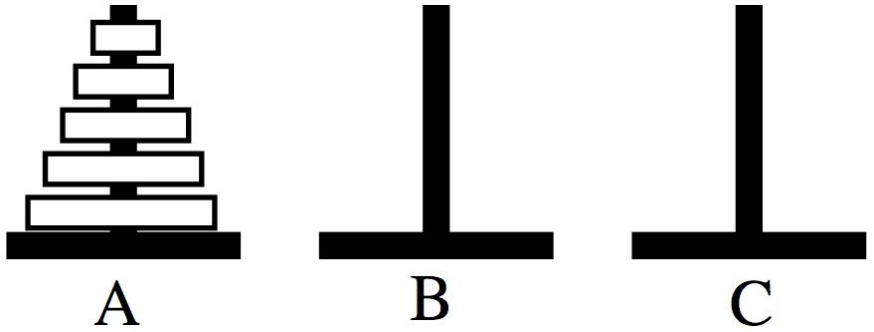
1. Make the legal move between A to B.
2. Make the legal move between A to C.
3. Make the legal move between B and C.
4. Repeat.



Tower of Hanoi

If the number of pegs is odd:

1. Make the legal move between A to C.
2. Make the legal move between A to B.
3. Make the legal move between B and C.
4. Repeat.



Tower of Hanoi

- Using these strategies, find the minimum number of moves to solve the puzzle with 3, 4, and 5 disks.

- Can you predict how many moves will it take to solve the puzzle with 10 disks?

Tower of Hanoi

Number of Disks	Number of Moves
1	1
2	3
3	
4	
5	
10	
n	

Tower of Hanoi

Number of Disks	Number of Moves
1	1
2	3
3	7
4	15
5	31
10	
n	

Tower of Hanoi

Number of Disks	Number of Moves
1	1
2	3
3	7
4	15
5	31
10	1023
n	2^n-1

Tower of Hanoi

Legend has it that a group of Eastern monks are the keepers of three towers on which sit 64 golden rings. Originally all 64 rings were stacked on one tower with each ring smaller than the one beneath. The monks are to move the rings from this first tower to the third tower one at a time but never moving a larger ring on top of a smaller one. Once the 64 rings have all been moved, the world will come to an end.

If the monks can move 1 ring per second, how many years would it take?

Tower of Hanoi

It would take the monks $2^{64} - 1 = 1.8446744 \times 10^{19}$ seconds, which equates to 586,549,402,018 years!

Brain Teaser

If everyone in this class shook everyone else's hand, how many handshakes would take place?

Brain Teaser

Number of People	Number of Handshakes
1	0
2	1
3	
4	
5	
n	

Brain Teaser

Number of People	Number of Handshakes
1	0
2	1
3	3
4	6
5	10
n	

Brain Teaser

Number of People	Number of Handshakes
1	0
2	1
3	3
4	6
5	10
n	$n(n+1)/2$

Certificates and Evaluation

1. Fill out the evaluation for Duke TIP.
2. Write down what you liked and didn't like about the class on a blank sheet of paper.
3. Receive your certificate.
4. Celebrate!